A Learnability Analysis on Neuro-symbolic Learning

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NEURAL INFORMATION PROCESSING SYSTEMS

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Neuro-Symbolic Learning

Neuro-symbolic learning (NeSy) unifies data-driven learning with knowledge-based reasoning, marking a ***paradigm shift*** often regarded as the ***third wave of AI***. Its ability to incorporate formal knowledge makes it especially significant for ***high-stakes*** domains such as law, medicine, and finance.

Usually, the training process of NeSy system is weakly-supervised manner, i.e., learning model f with only $(\boldsymbol{x}, y)$ pairs and background knowledge $\texttt{KB}$ to be satisfied.

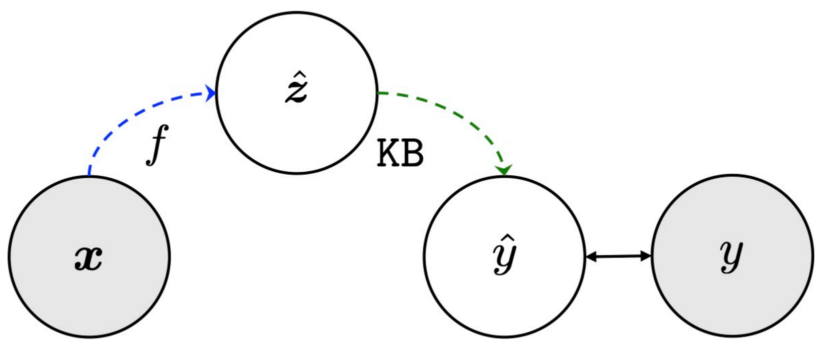

Figure 1: A typical inference process of neuro-symbolic system.

The objective is to learn a model $f : \mathcal{X} \to \mathcal{Z}$ that generalizes on concepts:

$$R_{0/1} = \mathbb{E}_{(x,z)}[\mathbb{I}(f(x) \neq z)]. \quad (1)$$

However, in the absense of supervision $\boldsymbol{z}$, only the surrogate is accessible:

$$R_{\mathrm{NeSy}}(f) = \mathbb{E}_{(\boldsymbol{x},y)}[\mathbb{I}(f(\boldsymbol{x}) \wedge \texttt{KB} \not\models y)]. \quad (2)$$

Learnability

Under what conditions can concept risk (1) be minimized through empirical risk minimization over NeSy risk (2) as sample size $\to \infty$?

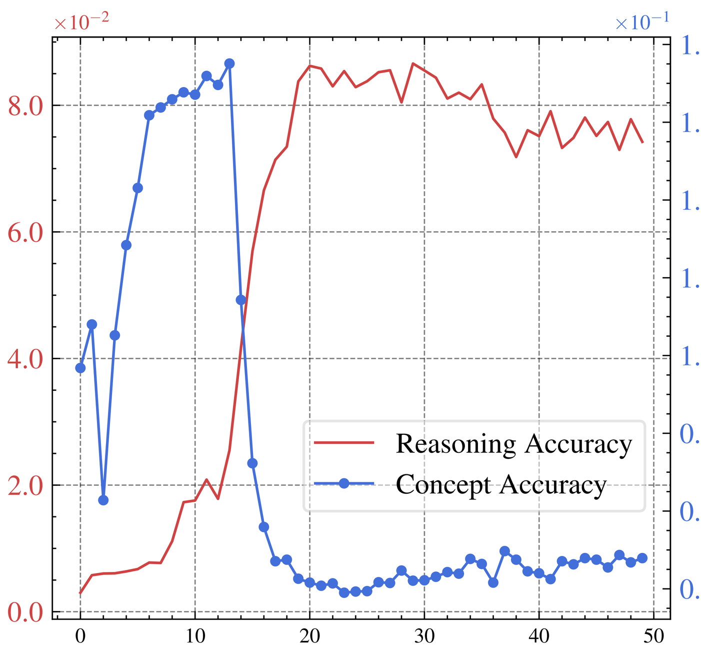

Figure 2: An unlearnable case

Motivation: Determining the learnability of a NeSy task is crucial for understanding how a NeSy system works and for inspiring the development of more principled and powerful algorithms.

Reasoning Shortcut: [1] identified the *RS problem* — models achieve high training likelihood while deviating from true concept distributions. While approaches exist to address RSs [2,3], a rigorous theoretical framework connecting RSs to statistical learnability remains underexplored.

Learnability is decided by DCSP solution space

- If the derived constraint satisfaction problem has a ***unique*** solution, the task is ***learnable***.
- Otherwise, the task is ***unlearnable***, the expected concept risk is bounded by the ***disagreement*** degree among solustions.

NeSy Task

SumMod10 (0, 1) = 1
…
SumMod10 (1, 9) = 0
…
SumMod10 (2, 9) = 1

Modular Addition
$y = (z_1 + z_2) \bmod 10$

→

Derived CSP

```
base = 10

# Create integer variables
X₀, X₁, …, X₉ ∈ [0, k-1]

# Constraint 1: all Xᵢ are distinct
all_different(X₀, X₁, …, X₉)

# Constraint 2: modular addition rules
for each pair (i, j) with i < j:
    (Xᵢ + Xⱼ) mod base = (i + j) mod base
```

→ Solve

DCSP Solutions

```
# Two solutions
# [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
sol1 = {X₀:0, X₁:1, …, X₉:9}

# [5, 6, 7, 8, 9, 0, 1, 2, 3, 4]
sol2 = {X₀:5, X₁:6, …, X₉:4}

# Disagreement among sol1 and sol2
d = disagreement(sol1, sol2) # 10
```

→

Learnability

Unlearnable

$\mathcal{E}^* \leq d/L = 10/10 = 1$

Figure 3: An example of learnability decision procedure.

Formal Description

Theorem: Given a NeSy task $\mathcal{T}$, the learnability is decided by the solution space of DCSP. Especially:

- If the DCSP has unique solution, $\mathcal{T}$ is learnable, and the concept risk $R_{0/1} \leq \varepsilon$ when the sample size

$$N \geq \frac{1}{\kappa} \cdot \log(|\mathcal{B}|/\varepsilon) \quad (3)$$

- Otherwise, $\mathcal{T}$ is unlearnable, the expected concept risk

$$\mathcal{E}^* \leq d/L \quad (4)$$

Implications

- Expected concept risk depends on the ***disagreement*** among DCSP solutions rather than #solutions (#RSs)
- Unlearnable NeSy tasks can be combined to be learnable (if their combined DCSP has unique solution).

Experiments

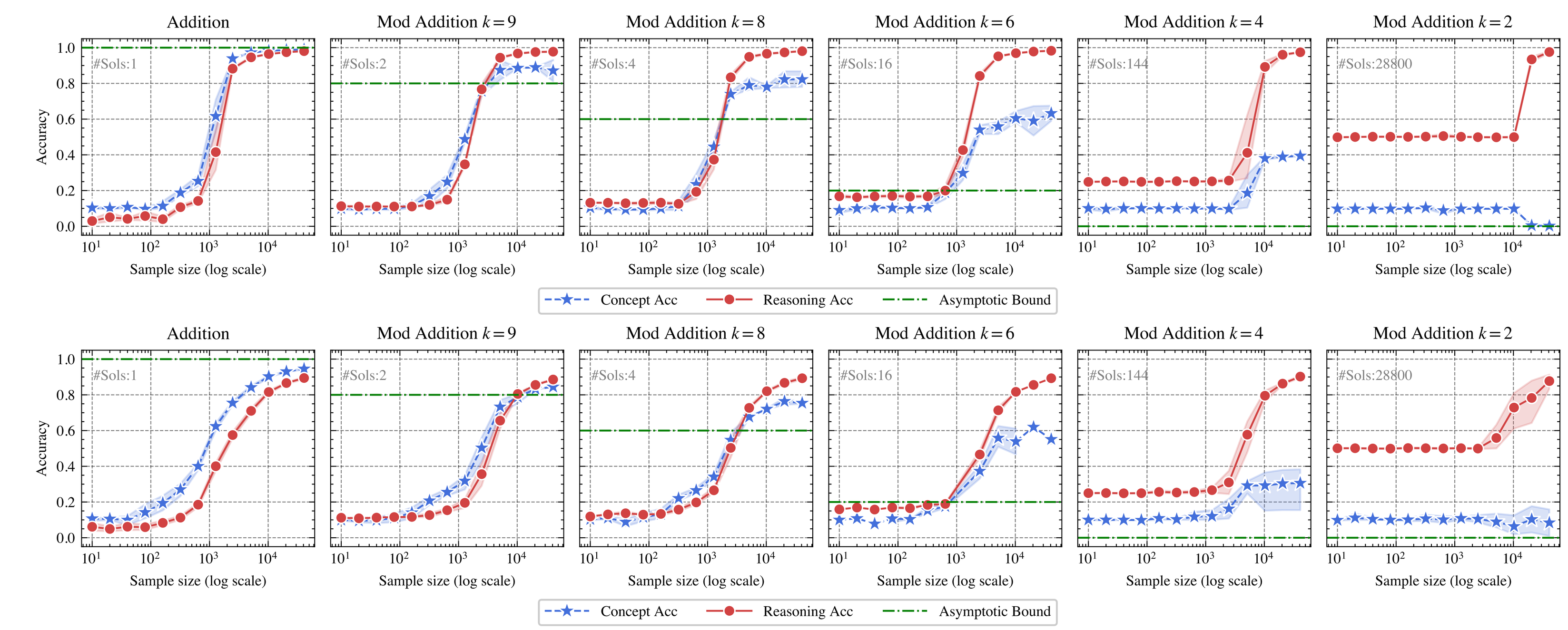

Figure 4: Accuracies vesus sample size.

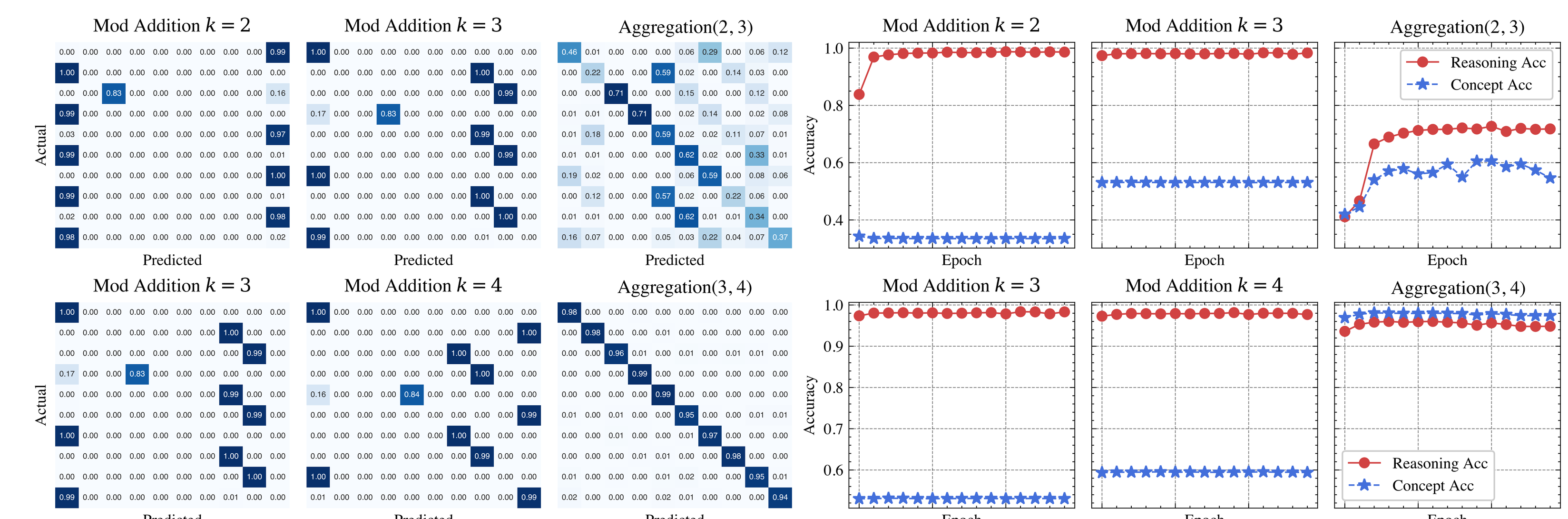

Figure 5: Combination of unlearnable NeSy tasks.

References

1. Marconato E, Teso S, Vergari A, Passerini A. Not All Neuro-Symbolic Concepts Are Created Equal: Analysis and Mitigation of Reasoning Shortcuts. In Advances in Neural Information Processing Systems. Curran Associates, Inc.; 2023. pp. 72507–39.
2. Marconato E, Bontempo G, Ficarra E, Calderara S, Passerini A, Teso S. Neuro-Symbolic Continual Learning: Knowledge, Reasoning Shortcuts and Concept Rehearsal. In Proceedings of the 40th International Conference on Machine Learning. Honolulu, Hawaii, USA: JMLR.org; 2023. pp. 23915–36.
3. Marconato E, Bortolotti S, Krieken E van, Vergari A, Passerini A, Teso S. BEARS Make Neuro-Symbolic Models Aware of their Reasoning Shortcuts. In Proceedings of the 40th Conference on Uncertainty in Artificial Intelligence. PMLR; 2024. pp. 2399–433.

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